Aeroservoelasticity in the Time Domain

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The objective of this investigation was to develop and evaluate a nonlinear technique for the stability assessment of high-performance aircraft in the time domain. The technique employed is compatible with popular digital flutter and/or vibration mathematical models and is adaptable to various aircraft control systems and/or modal information. The aircraft may possess servocontrol systems of high complexity and may dictate the use of aerodynamic schemes of the user's choice. The method of approach involves the use of the Advanced Continuous Simulation Language (ACSL), chosen because of its hybrid computer-like features and its conversational language. All modules of the simulation are solved in the time domain as the vehicle is subjected to disturbances of the user's choice. The vehicle motion rates are then output in the time domain for the engineer's concluding assessment of stability. Instabilities experienced by an early version of the F-16 aircraft are used as check cases. The stability boundary obtained is unconservative in its agreement with that determined in flight test, whereas the frequencies obtained are in excellent agreement. Conclusions include the following: 1) the simultaneous solution of large systems of aerodynamic, inertial, elastic, and servocontrol equations can be obtained using advanced simulation languages with mnemonic features that aid in their implementation and use; and 2) some improvements, when compared to frequency domain results, can be obtained using this nonlinear time domain simulation approach, especially when studying the behavior of phenomena with nonlinear effects or response.

Nomenclature

A	= tensor or matrix of inertia terms; see Eq. (20)
a	= acceleration
B	= direction cosines
b	= reference length for lateral equations
C	= stability derivatives, conventional subscripts
$\bar{c}$	= mean chord
D	= collection of I terms; see Eq. (20)
F	= force
f	= function to be searched
$\{G\}$	= generalized force vector
g	= gravitational constant
H	= angular momentum
h	= altitude
I	= inertia terms
J	= integer used in search algorithm
$[K]$	= stiffness matrix
L,M,N	= body moments analogous to the three components of the vector M_i
$[M]$	= mass matrix
M	= Mach number
m	= mass
$\{P\}$	= load vector
p,q,r	= roll, pitch, and yaw rates; see Fig. 1
Q	= intermediate vector; see Eq. (14)
$\bar{q}$	= dynamic pressure, $\frac{1}{2}\rho V^2$
$\{q\}$	= normal coordinates
R	= position vector
S	= reference area, wing planform
s	= Laplace transform variable
T	= thrust
t	= time
u,v,w	= velocities along x,y,z axes
x,y,h	= positional coordinates
X,Y,Z	= force components
x'	= variable for aerodynamic functions
V	= velocity
$\Rightarrow$	= stands for, or implies
α	= angle of attack
β	= sideslip angle
δ_{ij}	= Kronecker delta
δ	= control surface deflection
ϵ_{ijk}	= permutation symbol
ϕ	= roll angle
$[\Phi]$	= mode shape matrix
ψ	= yaw angle
θ	= pitch angle
ω_i	= rotation rate vector
ω	= circular natural frequency

Subscripts

a	= aileron
B	= body-fixed axes
c.m.	= center of mass
E	= Euler (inertial) axes
F	= flaperon
HT	= horizontal tail
i,j,k	= tensor notation indices
L,R	= left, right
N	= normal
p	= center-of-mass property
R	= rudder
S	= stability axes

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W = wind axes
x,y,z = axes reference
y = lateral
0 = initial value

Superscripts

-1 = inverse
T = transpose
$(\ )'$ = intermediate values

Introduction

TIME domain aeroelastic and/or aeroservoelastic investigations of flight vehicles have been the subject of considerable interest in the recent literature. From the large number of related works, only five journal articles of a specific nature will be mentioned here. Ballhaus and Georgian[1] used an indicial approach and LTRAN2 to produce some simple aeroelastic solutions with the emphasis on the aerodynamics involved. Swain and Yen[2] addressed the interaction of the pilot and the pitching time history as influenced by aeroelastic effects. Yoshino[3] emphasized a real-time longitudinal simulation which utilized an EAI 8400 computer in modeling some of the Boeing 707 response characteristics. Structural elasticity effects are included. Perry et al.[4] describe the use of DYLOFLEX in modeling flexible aircraft with an emphasis on calculating dynamic loads and incorporating active control systems. Yang and Chen,[5] in addition to reporting on transonic flutter analyses, obtained some neutrally stable time responses for various two- and three-degree-of-freedom (DOF) airfoils and used them to verify their flutter analyses. They also present a thorough survey of much of the research using LTRAN2 and related software in aeroelastic studies.

There are a number of books which address the fundamentals underlying time domain aeroservoelasticity; among them are those by Bisplinghoff and Ashley,[6] Vernon,[7] Blakelock,[8] Ashley,[9] and Dowell.[10] Ashley, especially, has in Ref. 9 (pp. 45-46) a succinct description of the difficulties involved. It is evident from an examination of these and related references that the complexities of the general problem are great, primarily because of the interaction of flexible bodies and their dynamic response at various speeds in a hostile environment and the large digital requirements for modeling of the equations governing this behavior.

The field of simulation of flight itself is represented excellently by the classic stability and control texts of Roskam,[11] Etkin,[12] McRuer et al.,[13] and others.

The incorporation of aircraft flexibility into flight simulations is at best difficult enough to 1) be either limited (Ref. 11, Chap. 8) or beyond the scope (Ref. 13, p. 683) of recent extensive treatises on control and flight dynamics; 2) cause the USAF analytical work in this field to be discontinued§ in 1962[14]; and 3) prompt statements throughout the literature such as the following:

> "Unfortunately, response calculations for systems with time-varying or non-linear elements present formidable difficulties...no general and established ways of solving..." (From Ref. 6, p. 488.)
>
> "Three significant phenomena that have been neglected... [includes treatment of flexibility]." (From Ref. 9, p. 41.)

On the other hand, significant (and difficult) approaches to the general problem have been addressed somewhat successfully by Roskam,[11] Schwanz,[14] Bisplinghoff and Ashley,[6] the Boeing Company,[15] and other researchers. Perhaps the briefest summary is given by Ashley[9] and one of the most comprehensive by Schwanz.[14]

§"Due to the absence of a digital computer system large enough to solve the complex equations developed using the structural and aerodynamic mathematical models that describe the physics of the elastic aircraft."

One of the most interesting aspects of the technical problem addressed herein is mentioned in Refs. 14 and 16. These comments concern interdisciplinary communication problems and the multiple disciplines involved in aeroservoelastic problems. They are a very important reason for the software approach taken herein. This approach involves the use of the Advanced Continuous Simulation Language (ACSL).¶ This is a very general and extensive simulation language that has advantages for special-purpose programs while retaining generality and ease of modification and, most importantly, having mnemonic and modular features which ease the development, maintenance, and use by those not totally trained in each aspect of the overall problem. This is not to imply ease of use by the novice, by any means; instead it means ease of use by the individual trained in stability and control, the structural dynamicist/aeroelastician, and the aerodynamicist, each of whom is assumed to be an individual knowledgeable in FORTRAN.

An approach using such an advanced simulation language has an appeal from other viewpoints. The most important of these is the trend toward more parallel-processing languages,[17,18] which indicates that future programming efforts will be less sequential in nature than FORTRAN and FORTRAN-like languages. This elimination of the need to do sequential-based programming is an ACSL advantage.

The objective of this effort is to utilize the YF-16 instabilities experienced in flight tests (detailed below) as check cases in the development of a modular computer simulation adaptable to changes so that the stability of the various aircraft of interest may be assessed in the time domain.

Equation Development

As evident upon perusal of the literature, approaches to flight simulation in stability and control, aeroelastic, and aeroservoelastic investigations are quite varied. In summary, studies in the time domain generally consist of one of the following: 1) one-degree-of-freedom roll, pitch, or yaw with or without elastic effects; 2) three lateral-directional equations, with or without elastic effects; 3) three longitudinal equations, with or without elastic effects; or 4) five- or six-degree-of-freedom equations with or without elastic effects.

Varied approaches to the aerodynamics involved would yield practically unlimited subclasses of these. The latter approach can itself be approached in various ways, the most popular probably being that involving traditional Euler angles. Quaternions are yet another approach. Alternatively, here, a direction cosine approach is taken, primarily due to the absence of singularity problems, but also due to the near one-to-one correspondence between the analytic development and current computer simulation terminology involving vector integrators which such notation affords.

The axis systems of interest are conventional body and stability axes. Figure 1 shows the direction cosines which locate the set of body axes with respect to a set of axes parallel to an inertial Euler axis system.

The complete system of equations which must be solved can be thought of in five categories: 1) computation of the body forces and moments; 2) resolution of forces to wind axes; 3) development of acceleration relationships; 4) resolution of velocity components into inertial axes; and 5) solution of integral relationships for velocities, rates, and position.

Solution of aeroservoelastic problems involving the complete elastic aircraft and control system requires accurate simulation of the six-degree-of-freedom flight equations governing the motion of the center of mass of the aircraft system. It has been noted[19] that the use of the flight path or

¶This simulation language is now available at over 250 installations around the world. It is extensively documented in the following: Advanced Continuous Simulation Language (ACSL) User Guide/Reference Manual, 3rd Ed., Mitchell and Gauthier Assoc., Inc., P.O. Box 685, Concord, Mass., 01742, 1981.

wind axis system is much more accurate and efficient than the body axis system from a computational standpoint for solving the translational equations of motion. For the rotational equations, the only reasonable coordinate system to use is that of the body axis. Otherwise, time-varying mass moments and products of inertia must be computed continuously for the entire system.

It is not the purpose of this paper to derive the complete set of flight equations; instead, they are collected and shown in Table 1. Interestingly, they are usually quite scattered in the available literature and are often incomplete. In Table 1, those equations presenting a choice between the use of Euler angles or direction cosines are shown side by side for clarity. The angles θ, ϕ, and ψ are the traditional Euler angles of Ref. 9, whereas B_{x_i}, B_{y_i}, B_{z_i} are used for direction cosines. The primed variables are intermediate expressions used to resolve body axis velocities into an inertial system. Also, note that in Table 1 the $\dot{p}_B$, $\dot{q}_B$, and $\dot{r}_B$ equations are for a symmetric aircraft for which the products of inertia vanish.

A tensor approach to the derivation of the general rotational rigid-body equations of motion is taken in the following paragraphs. Tensor notation and conventions[20] are used to simplify the derivations of subsequent programming.

Using the body-fixed system of Fig. 1, the aircraft's center-of-mass translational velocity and rotational velocity components can be written as

$$V_{\text{c.m.}_i} \Rightarrow (u_B, v_B, w_B) \tag{1}$$

$$\omega_i \Rightarrow (p_B, q_B, r_B) \text{ or } (\omega_1, \omega_2, \omega_3) \tag{2}$$

where the subscript B refers to the body axes.

The location of a differential element of mass in this system with respect to an Euler frame with origin at the center of gravity is

$$R_i \Rightarrow (x_B, y_B, z_B) \text{ or } (x_1, x_2, x_3) \tag{3}$$

The velocity of the element using the tensor form of the cross product is given by

$$V_i = V_{\text{c.m.}_i} + \epsilon_{ijk}\omega_j R_k \tag{4}$$

In the derivation of the rotational equations, the first term of Eq. (4) is neglected since it can be shown to vanish later. For this differential element of mass, the time rate of change of the angular momentum about the center of gravity is equal to the moment components M_i on the body; therefore,

$$M_i = \frac{\mathrm{d}H_i}{\mathrm{d}t} = \int_m \frac{\mathrm{d}}{\mathrm{d}t}(\epsilon_{ijk}R_j V_k \mathrm{d}m) \Rightarrow (L_B, M_B, N_B) \tag{5}$$

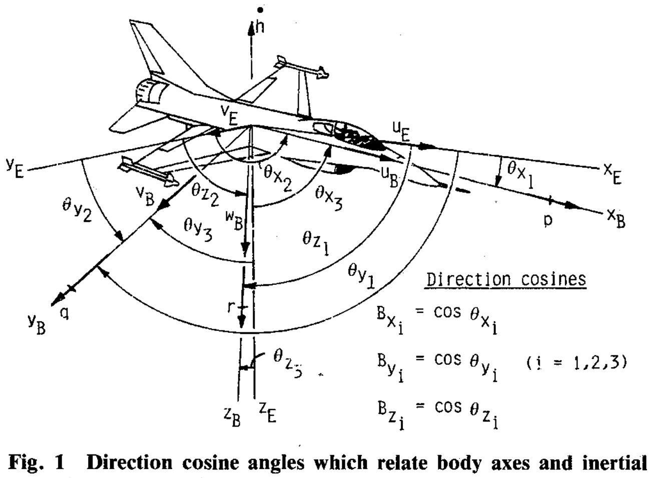

Fig. 1 Direction cosine angles which relate body axes and inertial axes. (The *E* subscript axes are a set parallel to an inertially fixed reference frame.)

The integrand in Eq. (5), when expanded, is

$$\epsilon_{ijk}\dot{R}_j V_k + \epsilon_{ijk}R_j\dot{V}_k + \epsilon_{ijk}\omega_j\epsilon_{krs}R_r V_s \tag{6}$$

If the element of mass is assumed fixed in the body, then $\dot{R} \Rightarrow (0,0,0)$ and the first term of Eq. (6) vanishes. For the second term,

$$\epsilon_{ijk}R_j\dot{V}_k = \epsilon_{ijk}R_j\epsilon_{krs}\dot{\omega}_r R_s \tag{7}$$

by virtue of the time derivative of Eq. (4).

Table 1 Collected equations of motion

Body forces and moments

$$L_B = \bar{q}SbC_l \qquad M_B = \bar{q}S\bar{c}C_m \qquad N_B = \bar{q}SbC_n$$

$$\begin{aligned} X_B &= T - \bar{q}SC_A - mg\sin\theta \\ Y_B &= \bar{q}SC_Y + mg\cos\theta\sin\phi \\ Z_B &= -\bar{q}SC_N + mg\cos\theta\cos\phi \end{aligned} \quad \text{or} \quad \begin{aligned} X_B &= T - \bar{q}SC_A + mg\cdot B_{x_3} \\ Y_B &= \bar{q}SC_Y + mg\cdot B_{y_3} \\ Z_B &= -\bar{q}SC_N + mg\cdot B_{z_3} \end{aligned}$$

Force resolution to wind axes

$$\begin{aligned} X_s &= X_B\cos\alpha + Z_B\sin\alpha \\ X_w &= X_s\cos\beta + Y_B\sin\beta \\ Y_w &= -X_s\sin\beta + Y_B\cos\beta \\ Z_w &= -X_B\sin\alpha + Z_B\cos\alpha \end{aligned}$$

Acceleration equations

$$\begin{aligned} \dot{V}_p &= X_w/m \\ \dot{\alpha} &= q_B - (p_B\cos\alpha + r_B\sin\alpha)\tan\beta + Z_w/(mV_p\cos\beta) \\ \dot{\beta} &= Y_w/(mV_p) - r_B\cos\alpha + p_B\sin\alpha \end{aligned}$$

$$\left.\begin{aligned} \dot{p}_B &= [L_B - q_B r_B(I_{zz} - I_{yy})]/I_{xx} \\ \dot{q}_B &= [M_B - p_B r_B(I_{xx} - I_{zz})]/I_{yy} \\ \dot{r}_B &= [N_B - p_B q_B(I_{yy} - I_{xx})]/I_{zz} \end{aligned}\right\} \text{See Eq. (21) for more general form}$$

$$\begin{aligned} \dot{\psi} &= (r_B\cos\phi + q_B\sin\phi)/\cos\theta \\ \dot{\theta} &= q_B\cos\phi - r_B\sin\phi \\ \dot{\phi} &= p_B + \dot{\psi}\sin\theta \end{aligned} \quad \text{or} \quad \begin{aligned} \dot{B}_{x_i} &= r_B B_{y_i} - q_B B_{z_i} \\ \dot{B}_{y_i} &= p_B B_{z_i} - r_B B_{x_i} \\ \dot{B}_{z_i} &= q_B B_{x_i} - p_B B_{y_i} \end{aligned}$$

Velocity resolution (to inertial axes)

$$\begin{aligned} u_B &= V_p\cos\alpha\cos\beta \\ v_B &= V_p\sin\beta \\ w_B &= V_p\sin\alpha\cos\beta \end{aligned}$$

$$\begin{aligned} w' &= v_B\sin\phi + w_B\cos\phi \\ v' &= v_B\cos\phi - w_B\sin\phi \\ u' &= u_B\cos\theta + w'\sin\theta \\ u_E &= u'\cos\psi - v'\sin\psi \\ v_E &= u'\sin\psi + v'\cos\psi \\ \dot{h} &= u_B\sin\theta - w'\cos\theta \end{aligned} \quad \text{or} \quad \begin{aligned} u_E &= u_B B_{x_1} + v_B B_{y_1} + w_B B_{z_1} \\ v_E &= u_B B_{x_2} + v_B B_{y_2} + w_B B_{z_2} \\ \dot{h} &= -(u_B B_{x_3} + v_B B_{y_3} + w_B B_{z_3}) \end{aligned}$$

Integrals

$$\begin{aligned} V_p &= V_{p_0} + \int\dot{V}_p\mathrm{d}t \\ \alpha &= \alpha_0 + \int\dot{\alpha}\mathrm{d}t \\ \beta &= \beta_0 + \int\dot{\beta}\mathrm{d}t \\ p_B &= p_{0_B} + \int\dot{p}_B\mathrm{d}t \\ q_B &= q_{0_B} + \int\dot{q}_B\mathrm{d}t \end{aligned} \qquad \begin{aligned} r_B &= r_{0_B} + \int\dot{r}_B\mathrm{d}t \\ x_E &= x_{0_E} + \int u_E\mathrm{d}t \\ y_E &= y_{0_E} + \int v_E\mathrm{d}t \\ h &= h_0 + \int\dot{h}\mathrm{d}t \\ M &= V_p/a \end{aligned}$$

$$\begin{aligned} \psi &= \psi_0 + \int\dot{\psi}\mathrm{d}t \\ \theta &= \theta_0 + \int\dot{\theta}\mathrm{d}t \\ \phi &= \phi_0 + \int\dot{\phi}\mathrm{d}t \end{aligned} \quad \text{or} \quad \begin{aligned} B_{x_i} &= \int\dot{B}_{x_i}\mathrm{d}t + B_{x_i}(0) \\ B_{y_i} &= \int\dot{B}_{y_i}\mathrm{d}t + B_{y_i}(0) \\ B_{z_i} &= \int\dot{B}_{z_i}\mathrm{d}t + B_{z_i}(0) \end{aligned}$$

Using Eqs. (4) and (7) and the right-most forms of Eqs. (2) and (3), Eq. (6) reduces to

$$\epsilon_{ijk}x_j\epsilon_{krs}\dot{\omega}_r x_s + \epsilon_{ijk}\omega_j\epsilon_{krs}x_r\epsilon_{stu}\omega_t x_u \tag{8}$$

It can now be shown that, by using the identity $\epsilon_{ijk}\epsilon_{krs} = \delta_{ir}\delta_{js} - \delta_{is}\delta_{jr}$ and the properties of the Kronecker delta, Eq. (8) may be arranged as

$$(\dot{\omega}_i x_j x_j - \dot{\omega}_j x_i x_j) + \epsilon_{ijk}\omega_j[\omega_k x_r x_r - \omega_r x_k x_r] \tag{9}$$

Now, substituting Eq. (9) for the integrand of Eq. (5), integrating over the mass of the body, and introducing the inertia tensor,

$$I_{ij} \equiv \int_m (x_s x_s \delta_{ij} - x_i x_j)\,\mathrm{d}m \tag{10}$$

gives the following expanded form of Eq. (5):

$$M_i = \frac{\mathrm{d}H_i}{\mathrm{d}t} = \dot{\omega}_m I_{mi} + \epsilon_{ijk}\omega_j(\omega_n I_{nk}) \tag{11}$$

The more traditional vector form of Eq. (11) is

$$M = \dot{\omega}\cdot\bar{\bar{I}} + \omega\times(\omega\cdot\bar{\bar{I}}) \tag{12}$$

Expanding Eq. (11), the complete rotational equations are

$$L_B = \dot{p}I_{xx} + \dot{q}I_{xy} + \dot{r}I_{xz} + q[pI_{zx} + qI_{zy} + rI_{zz}] - r[pI_{yx} + qI_{yy} + rI_{yz}] \tag{13a}$$

$$M_B = \dot{p}I_{yx} + \dot{q}I_{yy} + \dot{r}I_{yz} + r[pI_{xx} + qI_{xy} + rI_{xz}] - p[pI_{zx} + qI_{zy} + rI_{zz}] \tag{13b}$$

$$N_B = \dot{p}I_{zx} + \dot{q}I_{zy} + \dot{r}I_{zz} + p[pI_{yx} + qI_{yy} + rI_{yz}] - q[pI_{xx} + qI_{xy} + rI_{xz}] \tag{13c}$$

where the B subscript is implied for each p, q, and r.

Equations (13) are very inconveniently coupled in $\dot{p}$, $\dot{q}$, and $\dot{r}$, the roll, pitch, and yaw accelerations, respectively. They may be uncoupled, however, by defining a first-order tensor (vector) in terms of the left side and the right-most terms of Eq. (11). Thus, let

$$Q_i \equiv M_i - \epsilon_{ijk}\omega_j(\omega_n I_{nk}) = \dot{\omega}_m I_{mi} \tag{14}$$

Now, define the tensor A_{ij} such that

$$A_{ij}I_{jk} = \delta_{ik} \tag{15}$$

Note that in conventional matrix algebra A is the inverse of I, and the Kronecker delta δ_{ik} is the identity matrix.

Postmultiplying the terms of interest in Eq. (14) yields

$$\dot{\omega}_m I_{mk} A_{ki} = Q_k A_{ki} \tag{16}$$

Thus,

$$\dot{\omega}_m \delta_{mi} = Q_k A_{ki} \tag{17}$$

and, finally

$$\dot{\omega}_i = Q_k A_{ki} \tag{18}$$

After considerable algebra, the required elements for Q_k and A_{ki} are

$$Q_1 = L_B - (q^2 - r^2)I_{yz} - pqI_{xz} + prI_{xy} + qr(I_{yy} - I_{zz}) \tag{19a}$$

$$Q_2 = M_B - (r^2 - p^2)I_{xz} - qrI_{xy} + pqI_{yz} + pr(I_{zz} - I_{xx}) \tag{19b}$$

$$Q_3 = N_B - (p^2 - q^2)I_{xy} - prI_{yz} + qrI_{xz} + pq(I_{xx} - I_{yy}) \tag{19c}$$

where subscript B is implied for each p, q, and r; and

$$A_{ki} \Rightarrow \frac{1}{D}\begin{bmatrix} I_{yy}I_{zz} - I_{yz}^2 & & \\ I_{xz}I_{yz} - I_{xy}I_{zz} & I_{xx}I_{zz} - I_{xz}^2 & \\ I_{xy}I_{yz} - I_{xz}I_{yy} & I_{xy}I_{xz} - I_{yz}I_{xx} & I_{xx}I_{yy} - I_{xy}^2 \end{bmatrix} \tag{20}$$

with $D = I_{xx}I_{yy}I_{zz} + 2I_{xy}I_{yz}I_{xz} - I_{xx}I_{yz}^2 - I_{yy}I_{xz}^2 - I_{zz}I_{xy}^2$. Therefore, using Eqs. (18-20), the uncoupled equations for arbitrarily shaped bodies are

$$\dot{p}_B = Q_1A_{11} + Q_2A_{21} + Q_3A_{31} \tag{21a}$$

$$\dot{q}_B = Q_1A_{12} + Q_2A_{22} + Q_3A_{32} \tag{21b}$$

$$\dot{r}_B = Q_1A_{13} + Q_2A_{23} + Q_3A_{33} \tag{21c}$$

Aerodynamic Forces and Moments

No attempt was made in this effort to develop sophisticated analytical aerodynamics. Judged more important were: 1) that the programming effort of this work incorporate aerodynamics in a modular fashion so that the results of other aerodynamic efforts could be easily incorporated in the future, and 2) that the aerodynamics used would be consistent with the checkout case involved so that emphasis could be placed on other aspects of the simulation. With these goals in mind, the aerodynamics involved in the simulation of this effort are based upon Refs. 21 and 24.

For the test aircraft, the general aerodynamic forces and moments included in the body force and moment equations in Table 1 are highly nonlinear functions of several variables. These aerodynamic forces and moments are input as data tables in coefficient form. The force coefficients are normal force C_N, axial force C_A, and side force C_Y. The moment coefficients are rolling moment C_l, pitching moment C_m, and yawing moment C_n.

The data tables are taken directly from Ref. 21. Variables for the coefficients include Mach number; angle of attack; angle of sideslip; roll, pitch, and yaw rates; rates of change of angle of attack; all control surface deflections; and altitude.

Unfortunately, the data in Ref. 21 are given in terms of lift and drag coefficients which must be converted to normal and axial force prior to input. To illustrate how a coefficient is calculated, the equation for the normal force coefficient is given by

$$C_N = C_{N_1} + C_{N_2}\left[\frac{\dot{q}\bar{c}}{2V}\right] + C_{N_3}\left[\frac{\dot{\alpha}\bar{c}}{2V}\right] \tag{22}$$

where

$$C_{N_1} = C_{N_1}(a, \bar{M}, h, \delta_N)$$

$$C_{N_2} = C_{N_2}(\alpha, \bar{M}, h) \qquad C_{N_3} = C_{N_3}(\alpha, \bar{M}, h)$$

Values for C_{N_1}, C_{N_2}, and C_{N_3} are interpolated in the tables using linear interpolation between two consecutive values. Rather than using a time-consuming table look-up routine to search through the tables and then interpolate, an algorithm is used to determine which two values are required based on the values of the independent variables. To illustrate the algorithm, consider a number of discrete values f_i of a function corresponding to discrete values x_i' of a variable. The interpolated value of f corresponding to an arbitrary x' is related by

$$f(x') = f_J + [(f_{J+1} - f_J)/\Delta x'](x' - x_J') \tag{23}$$

where the integer J is given by

$$J = \mathrm{INT}[(x' - x'_0)/\Delta x'] \qquad x'_0 \le x' \le x'_{J_{max}}$$
$$= 0 \qquad x' < x'_0$$
$$= J_{max} - 1 \qquad x' > x'_{J_{max}}$$

Similar algorithms can be constructed for multiple independent variables. Notice that the tabular values of f_i must, as in Ref. 21, be given at *equally spaced intervals* $\Delta x'$ on $(x'_0, x'_{J_{max}})$.

Density, Speed of Sound, and Gravity Simulation

Variations of density, speed of sound, and gravity with altitude can have significant effects in several areas of flight simulation. Functional approximations of these variables are calculated in two conventional subroutines which are easily incorporated into the ACSL simulation.

Program Arrangement

Now that considerable attention has been given to equation development, let us return briefly to the conceptual ideas concerning the overall programming arrangement. Were the goals of this effort only to solve the equations of Table 1, there would be little advantage to anything other than a FORTRAN approach with a good integration scheme. The aircraft of interest, however, have complicated control systems involving literally dozens of elements in a control system block diagram. As an example, the F-16 pitch channel has sensors and many other elements modeled as lead lags, real poles, complex poles, and various nonlinear functions. Each of these elements can be conversationally modeled in the ACSL language while still retaining the versatility and capability of concurrent FORTRAN usage. ACSL has language features which enable the engineer to model, for example, *each transfer function* of a control system with *one* instruction. Also, an entire portion of a complicated control system may be included in a MACRO and then invoked with *only one* statement.

Figure 2 is an illustration from the point of view of an entire aircraft's different control subsystems. In Fig. 2, each major part of the YF-16 control system is shown by a block or MACRO. The MACRO details are omitted; only the input and output of each is shown. The inputs on the left side into the various pitch, yaw, roll, etc., MACRO's are aircraft parameters from the dynamic equations. The outputs on the right side from the MACRO simulations are the control surface deflections. Different aircraft will change each MACRO's detailed simulations (as it will the aerodynamics) but will not change the basic dynamic equations, the overall simulation approach, or, in general, the input/output parameters of the control system modeling. The conversational modeling of these MACRO's and the tidy merging of various multidisciplinary notations and concepts are distinct advantages in the use of a continuous system modeling language.

The overall simulation approach can best be illustrated by examination of Fig. 3, which is a flow chart of the general program arrangement. Here, each block may represent numerous ACSL and/or FORTRAN instructions. It is convenient to think of them as an aero module, dynamics module, etc., as indicated. The diamond-shaped block at the bottom of the figure is done only at the end of the simulation, e.g., $t = t_{max}$. The other blocks outside of the double-line border are done only at time zero. All blocks within the double-line border are easily processed using ACSL for the interval $0 \le t \le t_{max}$. It should be emphasized that such a simulation is very analog-like in concept, but that it is entirely a digital computer program. Fundamental to such an arrangement are the following assumptions of this endeavor, listed in order of significance:

1) The capability of time domain stability assessment of different flight vehicles is the primary feature desired of the program.

2) Major aircraft information available is in large matrix form, i.e., a digitized model. Extensive use of already developed aerodynamics, inversion subroutines, etc., is desirable.

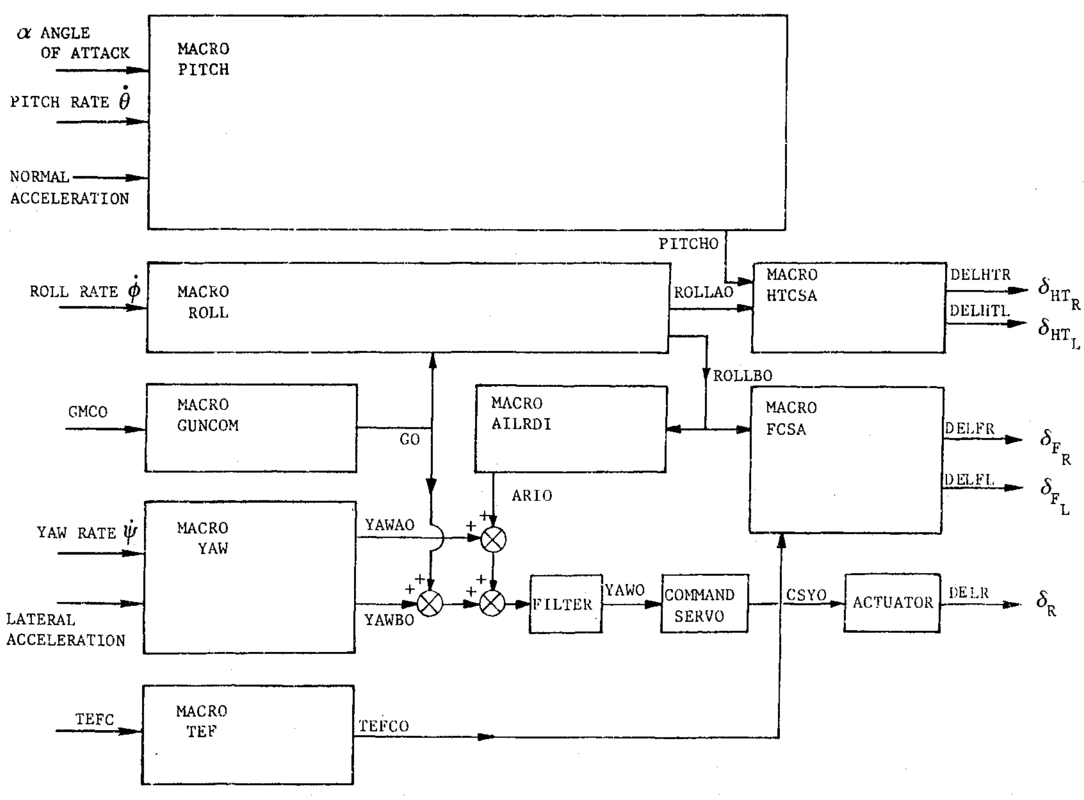

Fig. 2 YF-16 control system model MACRO's.

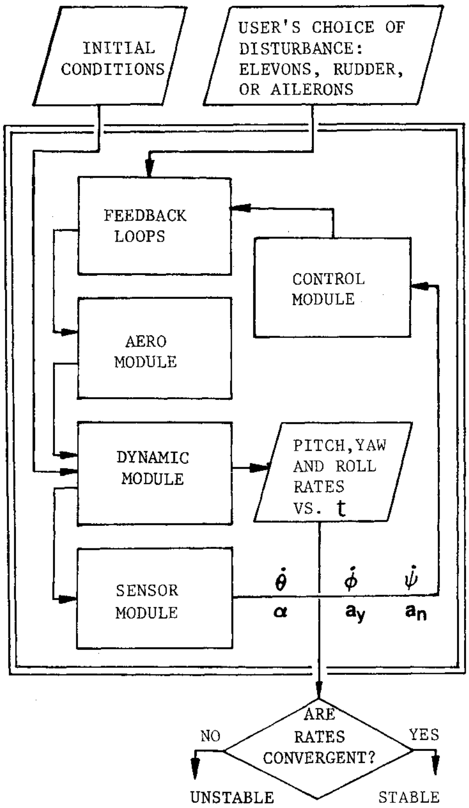

Fig. 3 Flow chart of aeroservoelastic time domain simulation.

3) The control system of the aircraft of interest is of significant complexity to merit detailed simulation and detailed interaction with other aspects of flight (aeroelastic, structural dynamics, etc.).

The ACSL language is very similar to NASTRAN®¶ in the concept that there is a *model definition* which is translated by the ACSL compiler, much like the *bulk data deck* in NASTRAN. This model is then controlled by *run-time* control instructions, again much like the *case control* instructions of NASTRAN.

Flexibility Effects

The major features involved in the incorporation of flexibility effects are described as follows:

1) The INITIAL section of the ACSL simulation is arranged to input a user-selected number of free vibration modes which has normally been generated from a vibrations and/or flutter analysis. This set of modes has associated with it a set of natural frequencies and a generalized mass matrix. The user then selects all or a lesser number n of these read-in modes to be used in the simulation.

2) This smaller set of digital information which describes the aircraft or missile model of interest is then transferred to a FORTRAN subroutine where the following is done:

a) Find the transpose of the phi (mode shape) matrix $(\Phi)^T$.

b) Form the generalized stiffness matrix according to

$$[K]=[M]\,[\lceil\omega^2\rfloor] \tag{24}$$

c) Invert the generalized mass matrix using a subroutine to form the matrix $[M]^{-1}$.

d) Multiply the inverse of the mass matrix by the stiffness matrix using a standard matrix-multiplying subroutine.

NOTE: steps 1 and 2 may be easily skipped on repeated runs by a SET RRFLAG=1 card in the run-time control instructions and a normal FORTRAN decision in the INITIAL section.

3) Then, in the time-stepping loop (the DYNAMIC section) with the control system simulation, form n differential equations involving the normal coordinates according to

$$\{\ddot{q}\}=[M]^{-1}[K]\{q\}+[M]^{-1}\{G\} \tag{25}$$

where $\{G\}$ is the vector of generalized forces given by

$$\{G\}=[\Phi]^T\{P\} \tag{26}$$

and $\{P\}$ is the vector of loads associated with flexing. The ability to determine the load vector $\{P\}$ is, of course, the subject of much current research.

4) Integrate these differential equations [Eq. (25)] to find the normal coordinate vectors $\{\dot{q}\}$ and $\{q\}$. Use is made of the ACSL vector integrators, INTVC, which provide, again, *one* instruction for *each* vector being integrated.

5) Use standard modal transformations to relate all structural points of interest with the normal coordinates (also for any derivatives). The column vectors and arrays required to accomplish these operations are easily incorporated into the ACSL simulation.

6) Use the coordinates and their derivatives obtained in step 5 to modify the rigid equations to include flexible effects, at least in some quasiexplicit manner.

Admittedly, these steps are difficult to do in a general manner, especially the difficulties in determining $\{P\}$ in Eq. (26) and in the modifications of step 6. Space does not permit detailed presentation of these matters here. The interested reader is referred to Ref. 22 for greater structural interaction detail.

Roll/Elastic Model

Although the major portion of this effort has been concerned with multiple degrees-of-freedom simulation, the flight instabilities incurred by the YF-16 during early testing are worthy of time domain simulation in a simpler manner. References 23-26 document some previous efforts. Since the YF-16 missiles-on instability involved primarily a rolling motion coupled with missile-pitching phenomena, it was decided to model this for check-case purposes with a roll model with flexibility effects. The missiles-off instability that was modeled rather well in Ref. 23 served as an excellent reference case.

Basically, the usual roll equation (Ref. 11, p. 446) is modified to include elastic effects and tip missile aerodynamic and inertia effects. Such an approach utilizes all of the blocks in the time-stepping loop of the simulation of Fig. 3 plus an additional one, an elastic module. Figure 4 shows a roll control loop for the YF-16, which includes a sensor, command servo, and actuator. A filter was added at a later time to prevent the instabilities.[24,25]

Simulation Results

Initially, several maneuvers were performed to assess the accuracy and adequacy of the full simulation. The steady-state maneuvers performed included a constant-altitude skid turn (unbanked), a barrel roll, and a loop. The roll and loop flight were made simply by deflecting and holding constant the aileron and elevator, respectively. The skid turn required a constant rudder deflection combined with a programmed aileron deflection to maintain zero roll angle. One of the advantages of the direction cosine approach is illustrated in the loop simulation where no singularity problems are encountered as the aircraft pitches through the vertical (±90-deg pitch angles). The barrel roll simulation did result in the aircraft attaining a steady-state roll rate, as it should.

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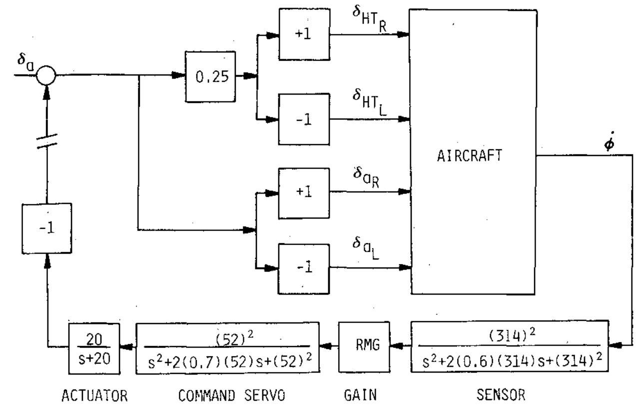

Fig. 4 Roll control loop for YF-16 aircraft.[24]

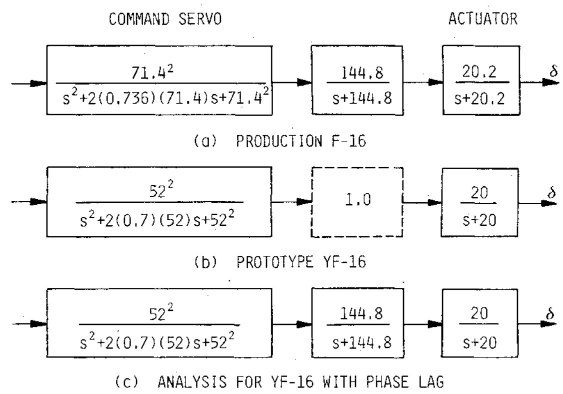

Fig. 6 Comparative aileron block diagram for missiles-off simulation studies.

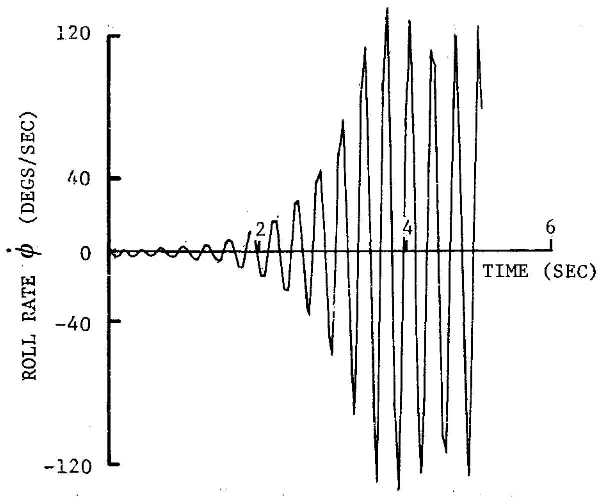

Fig. 5 Missiles-off instability simulation (Mach 0.98, altitude 15,000 ft).

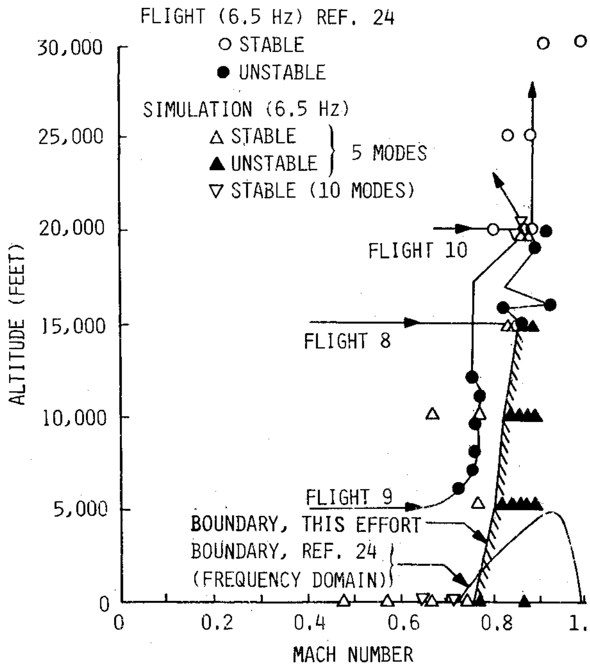

Fig. 7 Results of roll simulation with elastic effects, antisymmetric modes, and missiles-on.

Missiles-Off Instability

Accurate predictions of the missiles-off instability using the reference control system model and various aerodynamic characteristics have yet to be made. Several reasons for this failure have been postulated and investigated both in the literature and in the current work.[26] Peloubet et al.[24] state that there were two possible reasons for the inability of their analysis to predict instability: 1) there is an unknown increase in gain and/or phase lag somewhere in the roll loop; or 2) C_{l_δ} is larger than theory, wind-tunnel, and flight-test data indicate.

The second possibility was explored in their analysis, where, by using large values of C_{l_δ}, they were indeed able to predict an instability at about 4 Hz.

Moore, in a very detailed Laplace domain analysis,[27] artificially added a 20-deg phase lag to the roll loop. The instability that resulted matched flight-test data in both frequency and Mach number. Moore postulated that a phase lag existed and was due to shock motion on the control surface; i.e., the unsteady control surface aerodynamic model he used was not accurate enough. In effect, Moore and Peloubet verified that both possibilities 1 and 2 could produce the essentially rigid-body missiles-off instability. Moore's postulation is in line with the results of Peloubet and also seems to be a physically realizable case.

In the six-degree-of-freedom rigid-body time domain analysis of this work, with the complete YF-16 control system modeled, an artificial increase in C_{l_δ} produced an instability at 3.25 Hz. The general behavior of the instability, shown in Fig. 5, corresponds to that observed in flight tests. A linear transfer function analysis of the roll loop also showed that adjusting C_{l_δ} could produce a 3.9-Hz instability at the correct Mach number. Since the theoretical, wind-tunnel, and flight-test steady-state aerodynamics data agree reasonably well, the results of the above analyses raise the question of whether or not the dynamics of the control surface introduce a phase lag and gain in the roll loop, particularly when coupled with unsteady transonic aerodynamic phenomena. This possibility was tested by adding a block representing the unloaded aileron inertial response to the YF-16 system as shown in Fig. 6. With this term included, and using the flexible model wind-tunnel value for C_{l_δ} (slightly higher than the flight data value), the instability appeared at Mach 0.9 and 3.9 Hz. Based on this result and the analyses of Peloubet and Moore, it appears that the control surface dynamics coupled with a small increase in C_{l_δ} due to transonic effects caused the instability by introducing a phase lag and/or gain in the roll loop, and that the frequency is driven by the aerodynamic coefficients. (The phase lag and unsteady aerodynamics used in Moore's analysis gave the exact frequency.) Knowledge of the actuator torque and aileron inertial properties would be needed to explore in greater depth the suspected cause of the instability.

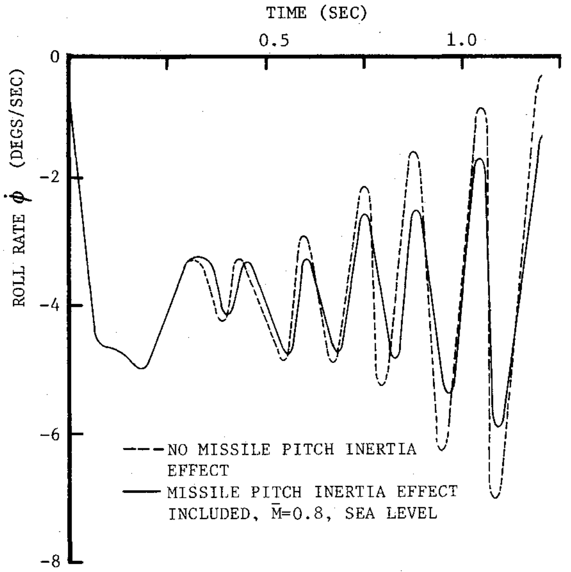

Fig. 8 Typical response of $\dot{\phi}$ during missiles-on instability.

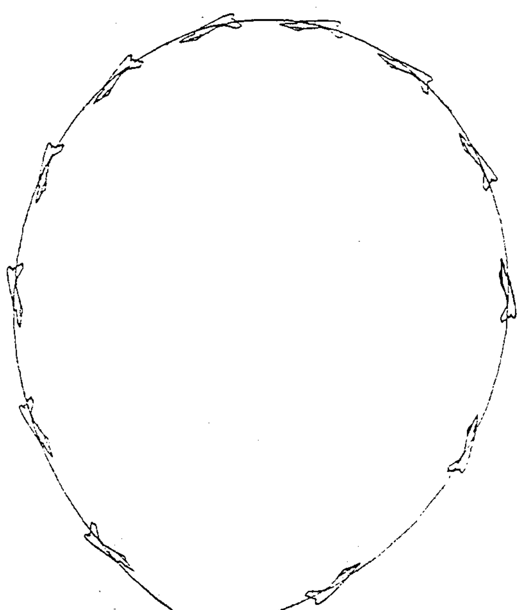

Fig. 9 Visual computer display of flight loop.

Missiles-On Instability

Results from the roll-only simulation, with elastic effects, are shown in Fig. 7. As can be seen, the simulation frequency and instability frequency observed in flight are identical. Although the stability boundary is still unconservative (as were frequency domain results), a stability boundary is predicted which follows the flight results more closely than did earlier efforts. As cited in the figure, these unstable points were obtained with five of the in-vacuum modes; increasing the number of modes used to 10 made little difference. Increasing C_{l_δ} moves the prediction points slightly closer to those observed in flight, consistent with statements in Refs. 24 and 27. A typical response of this roll-only simulation, with elastic effects, is shown in Fig. 8. Inclusion of the missile pitch rotational inertia does not seem to be as significant in affecting the instability as does the inclusion of the missile aerodynamic forces and the missile vertical translational inertia effects.

Simulation Visualization

Figure 9 shows a loop flight path with an F-16 side view superimposed at equal time intervals. Such a procedure helps in time domain visual simulation. It is possible that, once such motion data are obtained, display techniques could be utilized to provide real-time display visualization which would significantly aid the user in stability assessment. Also, the application to dynamic response studies is obvious.

Conclusions

The simultaneous solution of large systems of aerodynamic, inertial, elastic, and servocontrol equations can be obtained using advanced simulation languages with mnemonic features which aid in their implementation and use. Reasonable agreement in the time domain with an in-flight test case, missiles-on and missiles-off, has been obtained using aeroservoelastic simulations and quasisteady aerodynamics by roll only, roll only with elastic effects, and six degrees of freedom only. Results of a six-degree-of-freedom simulation with elastic effects will be the subject of a future paper. Although aerodynamic coefficient tables have been used herein, modular program design has been utilized to provide for easy replacement by other aerodynamic modules of the user's interest. Other aircraft, too, may be simulated by changing the various modules of the simulation. Although no claim is made that simulation is easily used, a user willing to become familiar with ACSL simulation has a powerful tool at his grasp—one which can result in simulation of the most sophisticated modern aircraft, limited only by the inadequacies of the particular module limitations.

The problem of machine time to real time exceeding 1 ($\simeq 5$) still exists, so that there remains the attractiveness of a hybrid computation. However, considering the computational speeds of the newer generations of computers, this may not be significant unless there are possible advantages of such simulations to real-time displays or tests.

Acknowledgments

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